Math 201 - Exam 2 (Fall 10)

T. Tlas

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- Please answer question 4 on the same sheet of paper on which it is written. Questions 2 and 3 have an extra sheet and question 1 has two extra sheets for you to write your answer on them. Any part of your answer written on the wrong page will not be graded.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 4 problems in total. Some questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$TOTAL$	

Problem 1 Consider the function of two independent variables

$$f(x, y) = (x^2 + y^2)^2 - 2x\,y$$

i- (10 points) What is the domain of the function? Is it bounded? Open? Closed? Give brief justifications to your answers.

ii- (10 points) Sketch the level curve of this function that passes through $(0, 0)$.

iii- (10 points) Find all the critical points of this function.

iv- (10 points) Find the plane tangent to the graph of this function for $(x, y) = (1, 1)$.

Hint: *You might find polar coordinates useful in part (ii).*

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

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Problem 2 (10 points each) Investigate which of the functions below possesses a limit as $(x, y) \to (0, 0)$. If the function has a limit find it, if not prove that it doesn't exist.

i-

$$f(x, y) = \frac{(x\,y)^2 + (x\,y)^4}{x^2 + y^2} + 1$$

ii-

$$g(x, y) = \frac{x\,y}{y - x^2 - x^3}$$

iii-

$$h(x, y) = \frac{x\,y}{y - x - x^3}$$

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER

Problem 3

(20 points) Which point(s) on the graph of the function $f(x, y) = xy + 2$ is nearest to the origin? In other words find the point(s) on the graph of this function whose distance to the origin is minimum.

Hint: *Distance is minimized if and only if the square of the distance is minimized.*

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ADDITIONAL SHEET FOR PROBLEM 3 ANSWER

Problem 4

(10 points) Assume $f(x, y)$ is a function whose range is $[0, \infty]$, and that for every c, the level curve corresponding to c is a square centered at $(0, 0)$ whose sides are parallel to the axes and have length equal to c. Find all the points where this function takes its maximum and minimum values on the circle $x^2 + y^2 = 1$, as well as the actual values of the maximum and the minimum on this circle.

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